Chapter 2. The electron theory

1.0. Introduction

The possibility of the formal representations of the Schreudinger and the Dirac electron equations in the form of the linear Maxwell equations was mentioned in several articles and books (Archibald, 1955; Akhiezer and Berestetskii, 1965; Koga, 1975; Campolattoro, 1980; Rodrigues, 2002).

According to postulate 6, an electromagnetic wave, which move along the closed curvilinear trajectory, must create the stabile objects that correspond to elementary particles of different kind.

Let's now translate this supposition into the mathematics language and show that in the simplest case the matrix form of the equations of such curvilinear waves mathematically fully coincides with quantum equations of vector and spinor (semi-vector) particles and gives many interesting consequences, which supplement the quantum field theory results.

2.0. Linear EM wave equation in the matrix form

We define as "linear" wave the solution of the linear wave equation.

Let us consider the plane-polarized linear electromagnetic (EM) wave moving, for example, on y - axis (fig.1):

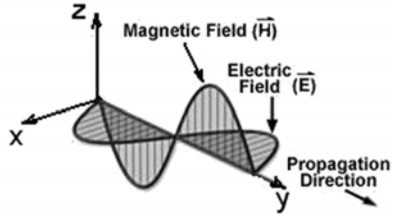

Fig. 1

where the electric and magnetic fields can be written in the complex form as:

$$\begin{cases} \vec{\mathrm{E}} = \vec{\mathrm{E}}_o e^{-i(\omega t \pm ky)}, \\ \vec{\mathrm{H}} = \vec{\mathrm{H}}_o e^{-i(\omega t \pm ky)}, \end{cases} \tag{2.1}$$

The electromagnetic wave of any direction has two plane polarizations and contains only four field vectors; for example, in the case of *y*-direction we have:

$$\vec{\Phi}(y) = \{\mathrm{E}_x, \mathrm{E}_z, \mathrm{H}_x, \mathrm{H}_z\}, \tag{2.2}$$

and $\mathrm{E}_y = \mathrm{H}_y = 0$ for all transformations. Note in this connection that the Dirac bispinor has also four components.

The EM wave equation has the following known view (Jackson, 1999):

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \vec{\nabla}^2 \right) \vec{\Phi}(y) = 0, \tag{2.3}$$

where $\vec{\Phi}(y)$ is any of the above electromagnetic wave field vectors (2.2). In other words this equation represents four equations: one for each vectors of the electromagnetic field.

We can also write this equation in the following operator form:

$$\left(\hat{\varepsilon}^{\,2} - c^2 \hat{\vec{p}}^{\,2} \right) \Phi(y) = 0, \tag{2.4}$$

where $\hat{\varepsilon} = i\hbar \dfrac{\partial}{\partial\, t}, \hat{\vec{p}} = -i\hbar\vec{\nabla}$ are the operators of the energy and momentum correspondingly and Φ is some matrix, which consists four components of $\vec{\Phi}(y)$.

Taking into account that $\left(\hat{\alpha}_o \hat{\varepsilon}\,\right)^2 = \hat{\varepsilon}^{\,2}$, $\left(\hat{\vec{\alpha}}\;\hat{\vec{p}}\right)^2 = \hat{\vec{p}}^{\,2}$, where $\hat{\alpha}_0 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & \hat{\sigma}_0 \end{pmatrix}$; $\hat{\vec{\alpha}} = \begin{pmatrix} 0 & \hat{\vec{\sigma}} \\ \hat{\vec{\sigma}} & 0 \end{pmatrix}$; $\hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & -\hat{\sigma}_0 \end{pmatrix}$ are Dirac's matrices and $\hat{\sigma}_0, \hat{\vec{\sigma}}$ are Pauli matrices, the equation (2.4) can also be represented in the matrix form of the Klein-Gordon-like equation without mass:

$$\left[\left(\hat{\alpha}_o \hat{\varepsilon}\,\right)^2 - c^2 \left(\hat{\vec{\alpha}}\;\hat{\vec{p}}\right)^2 \right] \Phi = 0, \tag{2.5}$$

Taking also into account that in case of photon we have $\omega = \varepsilon / \hbar$ and $k = p / \hbar$, from (2.5), using (2.1), we obtain $\varepsilon = cp$, as it is has place for a photon. Therefore we can consider the Φ - wave function of the equation (2.5) both as EM wave and as a photon.

Factorizing (2.5) and multiplying it from left on the Hermitian-conjugate function Φ^+ we get:

$$\Phi^+ \left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}}\;\hat{\vec{p}} \right) \left(\hat{\alpha}_o \hat{\varepsilon} + c\hat{\vec{\alpha}}\;\hat{\vec{p}} \right) \Phi = 0, \tag{2.6}$$

The equation (2.6) may be disintegrated on two Dirac-like equations without mass:

$$\Phi^+ \left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}}\;\hat{\vec{p}} \right) = 0, \tag{2.7'}$$

$$\left(\hat{\alpha}_o \hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)\Phi = 0\,, \qquad (2.7'')$$

It is not difficult to show that only in the case when we choose the Φ -matrix in the following form:

$$\Phi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix},\ \Phi^+ = \left(E_x\ \ E_z\ \ -iH_x\ \ -iH_z\right), \qquad (2.8)$$

the equations (2.7) are the right Maxwell equations of the electromagnetic waves: retarded and advanced. Actually using (2.8) and putting it in (2.7) we obtain:

$$\begin{cases} \dfrac{1}{c}\dfrac{\partial E_x}{\partial t} - \dfrac{\partial H_z}{\partial y} = 0 \\ \dfrac{1}{c}\dfrac{\partial H_z}{\partial t} - \dfrac{\partial E_x}{\partial y} = 0 \\ \dfrac{1}{c}\dfrac{\partial E_z}{\partial t} + \dfrac{\partial H_x}{\partial y} = 0 \\ \dfrac{1}{c}\dfrac{\partial H_x}{\partial t} + \dfrac{\partial E_z}{\partial y} = 0 \end{cases}, \quad (2.9') \qquad \begin{cases} \dfrac{1}{c}\dfrac{\partial E_x}{\partial t} + \dfrac{\partial H_z}{\partial y} = 0 \\ \dfrac{1}{c}\dfrac{\partial H_z}{\partial t} + \dfrac{\partial E_x}{\partial y} = 0 \\ \dfrac{1}{c}\dfrac{\partial E_z}{\partial t} - \dfrac{\partial H_x}{\partial y} = 0 \\ \dfrac{1}{c}\dfrac{\partial H_x}{\partial t} - \dfrac{\partial E_z}{\partial y} = 0 \end{cases}, \quad (2.9'')$$

For waves of any other direction the same results can be obtained by the cyclic transposition of the indexes and by the canonical transformation of matrices and wave functions (see chapter 3).

We will further conditionally name each of (2.7) equations the linear semi-photon equations, remembering that it was obtained by division of one wave equation of a photon into two equations of the electromagnetic waves: retarded and advanced.

3.0. Twirl transformation of electromagnetic wave

The transformation of the linear wave to the curvilinear (briefly – “twirl transformation”) can be conditionally represented as following expression:

$$\hat{R}\Phi \to \Psi\,, \qquad (3.1)$$

where $\hat{R}$ is the operator of trajectory transformation of EM wave from linear to curvilinear, the Φ is the wave function, defined by matrix (2.8), which satisfies the equations (2.5) and (2.7), and Ψ is some wave function:

$$\Psi = \begin{pmatrix} \mathrm{E}'_x \\ \mathrm{E}'_z \\ i\mathrm{H}'_x \\ i\mathrm{H}'_z \end{pmatrix}, \tag{3.2}$$

which appears after non-linear transformation; here $\Psi(y) = \{\mathrm{E}'_x, \mathrm{E}'_z, \mathrm{H}'_x, \mathrm{H}'_z\}$ are electromagnetic field vectors after twirl transformation.

As it is known, the description of vector transition from linear to curvilinear trajectory is fully described by differential geometry (Eisenhart, 1960). Note also that mathematically this transition is equivalent to the vector transition from flat space to the curvilinear space, which is described by Riemann geometry.

In connection to this let us remind that the Pauli matrices as well as the photon matrices are the space rotation operators – 2-D and 3-D correspondingly (Ryder, 1987).

3.1. The twirl transformation description in differential geometry

Let the plane-polarized wave, which has the field vectors $(\mathrm{E}_x, \mathrm{H}_z)$, be twirled with some radius r_p in the plane (X', O', Y') of a fixed co-ordinate system (X', Y', Z', O') so that E_x is parallel to the plane (X', O', Y') and H_z is perpendicular to it (figs 1 and 2).

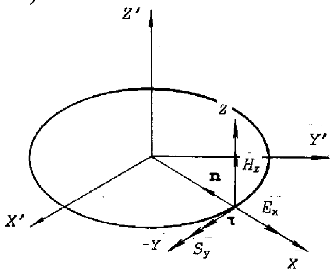

Fig. 2

According to Maxwell (Jackson, 1999) the displacement current in the equation (2.9) is defined by the expression:

$$j_{dis} = \frac{1}{4\pi} \frac{\partial \vec{\mathrm{E}}}{\partial t}, \qquad . \tag{3.3}$$

The above electrical field vector $\vec{\mathrm{E}}$, which moves along the curvilinear trajectory (let it have direction from the center), can be written in the form:

$$\vec{\mathrm{E}} = -\mathrm{E} \cdot \vec{n}, \tag{3.4}$$

where $\mathrm{E} = |\vec{\mathrm{E}}|$, and $\vec{n}$ is the normal unit-vector of the curve (having direction to the center). The derivative of $\vec{\mathrm{E}}$ can be represented as:

$$\frac{\partial \vec{\mathrm{E}}}{\partial t} = -\frac{\partial \vec{\mathrm{E}}}{\partial t}\vec{n} - \mathrm{E}\frac{\partial \vec{n}}{\partial t}, \tag{3.5}$$

Here the first term has the same direction as $\vec{\mathrm{E}}$. The existence of the second term shows that at the twirling of the wave the additional displacement current appears. It is not difficult to show that it has direction, tangential to the ring:

$$\frac{\partial \vec{n}}{\partial t} = -\upsilon_p \kappa \vec{\tau}\,, \tag{3.6}$$

where $\vec{\tau}$ is the tangential unit-vector, $\upsilon_p \equiv c$ is the electromagnetic wave velocity, $\kappa = \dfrac{1}{r_p}$ is the curvature of the trajectory and r_p is the curvature radius. Thus, the displacement current of the plane wave, moving along the ring, can be written in the form:

$$\vec{j}_{dis} = -\frac{1}{4\pi}\frac{\partial \mathrm{E}}{\partial t}\vec{n} + \frac{1}{4\pi}\omega_p \mathrm{E}\cdot\vec{\tau}\,, \tag{3.7}$$

where $\omega_p = \dfrac{m_p c^2}{\hbar} = \dfrac{\upsilon_p}{r_p} \equiv c\kappa$ we will name the curvature angular velocity, $\varepsilon_p = m_p c^2$ is photon energy, m_p is some mass, corresponding to the energy ε_p, $\vec{j}_n = \dfrac{1}{4\pi}\dfrac{\partial \mathrm{E}}{\partial t}\vec{n}$ and $\vec{j}_\tau = \dfrac{\omega_p}{4\pi}\mathrm{E}\cdot\vec{\tau}$ are the normal and tangent components of the current of the twirled electromagnetic wave, correspondingly. Thus:

$$\vec{j}_{dis} = \vec{j}_n + \vec{j}_\tau\,, \tag{3.8}$$

The currents $\vec{j}_n$ and $\vec{j}_\tau$ are always mutually perpendicular, so that we can write them in the complex form:

$$j_{dis} = j_n + i j_\tau\,, \tag{3.8'}$$

where $j_\tau = \frac{\omega_p}{4\pi} E$. Thus the tangent current appearance causes the appearance of imaginary unit. From the above we can also assume that *the appearance of imaginary unit in the quantum mechanics is tied with the tangent current appearance.*

3.2. The twirl transformation description in Rieman geometry

We can consider conditionally the Maxwell wave equations (2.7) with wave function (2.8) as Dirac equation without mass.

The generalization of the Dirac equation on the curvilinear (Riemann) geometry is connected with the parallel transport of the spinor in the curvilinear space (Fock, 1929a,b; Fock and Ivanenko, 1929; Van der Waerden, 1929; Schroedinger, 1932; Infeld und Van der Waerden, 1933; Goenner, 2004). For the generalization of the Dirac (without mass) equation on the Riemann geometry it is enough to replace the usual derivative $\partial_\mu \equiv \partial / \partial x_\mu$ (where x_μ are the co-ordinates in the 4-space) with the covariant derivative:

$$D_\mu = \partial_\mu + \Gamma_\mu , \tag{3.9}$$

where $\mu = 0, 1, 2, 3$ are the summing indices, and Γ_μ is the analogue of Christoffel's symbols in the case of the spinor theory, called Ricci symbols (or connection coefficients).

In the theory it shown that $\hat{\alpha}_\mu \Gamma_\mu = \hat{\alpha}_i p_i + i\hat{\alpha}_0 p_0$, where p_i and p_0 are the real values. It is not difficult to see that the tangent current j_τ corresponds to the Ricci connection coefficients (symbols) Γ_μ.

When a spinor moves along the straight line, all the symbols $\Gamma_\mu = 0$, and we have a usual derivative. But if a spinor moves along the curvilinear trajectory, not all the Γ_μ are equal to zero and a supplementary term appears.

Typically, the last one is not the derivative, but it is equal to the product of the spinor itself with some coefficient Γ_μ, which is increment in spinor. Since, according to the general theory (Sokolov and Ivanenko, 1952), the increment in spinor Γ_μ has the form and the dimension of the energy-momentum 4-vector, it is logical to identify Γ_μ with 4-vector of energy-momentum of the photon electromagnetic field:

$$\Gamma_\mu = \{\varepsilon_p, c\vec{p}_p\}, \tag{3.10}$$

where ε_p and p_p is the photon energy and momentum (not the operators). In other words we have:

$$\hat{\alpha}_\mu \Gamma_\mu = \hat{\alpha}_0 \varepsilon_p + \hat{\vec{\alpha}}\ \vec{p}_p\,, \qquad (3.11)$$

Taking into account that according to energy conservation law $\hat{\alpha}_0 \varepsilon_p + \hat{\vec{\alpha}}\ \vec{p}_p = \pm\hat{\beta}\ m_p c^2$, it is not difficult to see that the supplementary term contains a twirled wave mass.

4.0. The equations of twirled electromagnetic wave

4.1. Klein-Gordon-like equation of twirling photon

As it is follows from previous sections due to the curvilinear motion of the electromagnetic wave, some additional terms $K = \hat{\beta}\ m_p c^2$, corresponding to the tangent components of the displacement current, will appear in the equation (2.6), so that from (2.6) we have:

$$\left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}} \cdot \hat{\vec{p}} - K\right) \left(\hat{\alpha}_o \hat{\varepsilon} + c\hat{\vec{\alpha}} \cdot \hat{\vec{p}} + K\right) \Psi = 0\,, \qquad (4.1)$$

Thus, in the case of the curvilinear motion of the electromagnetic fields of photon, instead of the equation (2.6) we obtain the Klein-Gordon-like equation with mass (Schiff, 1955):

$$\left(\hat{\varepsilon}^2 - c^2 \hat{\vec{p}}^2 - m_p^2 c^4\right) \Psi = 0, \qquad (4.2)$$

As we see the Ψ -function, which appears after electromagnetic wave twirling and satisfies the equation (4.2), is not identical to the Φ -function before twirling. The Φ -function is the classical linear electromagnetic wave field, which satisfies the wave equation (2.7); in the same time the Ψ -function is the non-classical curvilinear electromagnetic wave field, which satisfies the Klein-Gordon-like equation (4.2).

As it is known in quantum physics the Klein-Gordon equation is considered as the scalar field equation. But obviously *the Klein-Gordon-like equation* (4.2), *whose wave function is* 4×1 *- matrix with electromagnetic field components, cannot have the sense of the scalar field equation.* Actually, let us analyze the objects, which this equation describes.

From the Maxwell equations follows that each of the components $E_x, E_y, E_z, H_x, H_y, H_z$ of vectors of an electromagnetic field $\vec{E}, \vec{H}$ submits to the same form of the scalar wave equations. In the case of the linear wave all field components are independent. Here by study of one of the $\vec{E}, \vec{H}$ vector's components, we can consider the vector field as scalar. But after the twirl transformation, i.e. in the framework CWED, when a tangential current appears, we cannot proceed to the scalar theory, since the components of a vector $\vec{E}$, as it follows from the condition (Maxwell law) $\vec{\nabla}\cdot\vec{E}=\frac{4\pi}{c}\vec{c}^{\,0}\cdot\vec{j}$ (where $\vec{c}^{\,o}$ is the unit vector of wave velocity) are not independent functions.

Therefore, although the Klein - Gordon equation for scalar wave function describes a massive particle with spin zero (spinless boson), the equations (4.2) concerning electromagnetic wave functions (3.2), which appears after curvilinear transformation, represents the equation of the vector particle with rest mass m_p and with spin one. In this sense this equation play the role of the Proca equation. To except this difficulty with the name we will name it as the **twirled photon equation**.

4.2. The equation of the twirled semi-photon

Using the disintegration (4.1) we can obtain from **twirled photon equation** (4.2) the equations of the twirled electromagnetic wave – advanced and retarded:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon}+c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)+\hat{\beta}\ m_pc^2\right]\psi=0, \qquad (4.3')$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon}-c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)-\hat{\beta}\ m_pc^2\right]=0, \qquad (4.3'')$$

where $\psi=\{E_x,E_z,H_x,H_z\}$ is some EM wave function, which we will be name further the **twirled semi-photon equations**. And the above transition from (4.2) to (4.3) we can conditionally name a “**symmetry breaking transformation**”.

Now we will analyse the particularities of the equations (4.3). It is not difficult to see that the lasts are similar to Dirac electron equations. But note that instead of electron mass m_e, equations (4.3) contain the twirled photon mass m_p. The question arises what type of EM particles the equations (4.3) describe?

In the case of electron-positron pair production it must be $m_p=2m_e$ so that from (4.3) we have:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon}+c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)+2\hat{\beta}\ m_ec^2\right]\psi=0, \qquad (4.4')$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon}-c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)-2\hat{\beta}\ m_ec^2\right]=0, \qquad (4.4'')$$

Obviously after the twirled photon breaking, i.e. after the chargeless twirled photon is divided into two charged semi-photon, the plus and minus charged particles acquire the electric fields, and each particle begins to move in the field of another. In order to become the independent (i.e. free) particles, they must be drawn away one from the other on great distance (fig.3):

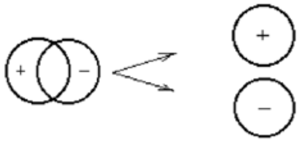

Fig.3

Therefore, the equations, which arise after the twirled photon division, cannot be the free positive and negative (electron and positron) particle equations, but the particle equations with the external field.

In this case during the charged particles move away one from another the energy, which correspond to the energy of the electric field creation, must be expended. In fact, being the particles combined, the system doesn't have any field (fig. 3). At very small distance they create the dipole field. And at a distance, much more than the particle radius, the plus and minus particles acquire the full electric fields. As it is known (Jackson, 1999), the potential V_P of two plus and minus charges in the point P is defined as (fig.4):

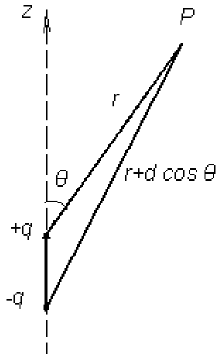

Fig. 4

$$V_P = \frac{e}{4\pi}\left(\frac{1}{r} - \frac{1}{r + d\cos\theta}\right), \tag{4.5}$$

where $\pm e$ are the dipole charges, d is the distance between the charges, and θ is the angle between axes and radius-vector of plus particle. For $d = 0$ we have $V_p = 0$. For $d \to \infty$ we obtain, as the limit case, the Coulomb potential for each free particles:

$$\lim_{d\to\infty} V_P = \frac{1}{4\pi}\frac{e}{r}, \tag{4.6}$$

Thus during the breaking process the particle charges appear. For the particle, removed to infinity, the work against the attractive forces needed to be fulfilled:

$$\varepsilon_{rel} = \oint eV_P d\upsilon = \frac{1}{4\pi} \oint \frac{e^2}{r} d\upsilon \,, \qquad (4.7)$$

Obviously, the external particles field defines this work, so that the release energy is the field production energy and in the same time it is the annihilation energy. Therefore, due to energy conservation law this energy value for each particle must be equal $\varepsilon_{rel} = m_e c^2$.

So, the equations (4.3) we can write in the following form:

$$\left[\left(\hat{\alpha}_0 \hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + \hat{\beta}\ m_e c^2 + \hat{\beta}\ m_e c^2\right]\psi = 0 \,, \qquad (4.8')$$

$$\psi^+\left[\left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) - \hat{\beta}\ m_e c^2 - \hat{\beta}\ m_e c^2\right] = 0 \,, \qquad (4.8'')$$

Using the linear equation for description of the energy conservation law, we can write:

$$\pm \hat{\beta}\ m_e c^2 = -\varepsilon_{ex} - c\hat{\vec{\alpha}}\ \vec{p}_{ex} = -e\varphi_{ex} - e\hat{\vec{\alpha}}\ \vec{A}_{ex} \,, \qquad (4.9)$$

where "*ex*" means "external". Putting (4.9) in (4.8) we obtain the Dirac equation with external field:

$$\left[\hat{\alpha}_0\left(\hat{\varepsilon} \mp \varepsilon_{ex}\right) + c\hat{\vec{\alpha}} \cdot \left(\hat{\vec{p}} \mp \vec{p}_{ex}\right) + \hat{\beta}\ m_e c^2\right]\psi = 0 \,, \qquad (4.10)$$

which at $d \to \infty$ give the Dirac free plus and minus particle equations:

$$\left[\left(\hat{\alpha}_o \hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + \hat{\beta}\ m_e c^2\right]\psi = 0 \,, \qquad (4.11')$$

$$\psi^+\left[\left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) - \hat{\beta}\ m_e c^2\right] = 0 \,, \qquad (4.11'')$$

From above some interesting consequences follow:

1. before breaking the twirled photon is not an absolutely neutral particle, but a dipole; therefore, it must have the dipole moment.

2. the formula (4.9) shows that in CWED the mass is not equivalent to the energy, but to the 4-vector of the energy-momentum; from this follows that in CWED the energy has the kinetic origin.

3. in framework of CWED for free term of particle equation the following expression take place:

$$\pm \hat{\beta}\ m_e c^2 = -\varepsilon_{in} - c\hat{\vec{\alpha}}\ \vec{p}_{in} = -e\varphi_{in} - e\hat{\vec{\alpha}}\ \vec{A}_{in} \,, \qquad (4.12)$$

where "*in*" means "internal". In other words the values $(\varepsilon_{in}, \vec{p}_{in})$ describe the inner field, and the values $(\varepsilon_{ex}, \vec{p}_{ex})$ the external field of electron-positron particles. When we consider the electron particle from great distance, the field

$(\varepsilon_{in}, \vec{p}_{in})$ works as the mass, and the term $(\varepsilon_{ex}, \vec{p}_{ex})$ describes the external electromagnetic field (and we have linear Dirac equations of particles). Inside the electron the term $(\varepsilon_{in}, \vec{p}_{in})$ is needed for the detailed description of the inner field of particle, which characterizes the particle parts interaction (as it is shown below, this term carries to non-linear equation of particle).

Using (3.2) we obtain electromagnetic form of the equations (4.11):

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = -ij_x^e \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = ij_z^m \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = -ij_z^e \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = ij_x^m \end{cases}, \quad (4.13') \qquad \begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = -ij_x^e \\ \frac{1}{c}\frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = ij_z^m \\ \frac{1}{c}\frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = -ij_z^e \\ \frac{1}{c}\frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = ij_x^m \end{cases}, \quad (4.13'')$$

where

$$\begin{aligned} j^e &= i\frac{\omega_e}{4\pi}E = i\frac{c}{4\pi}\frac{1}{r_e}E \\ j^m &= i\frac{\omega_e}{4\pi}H = i\frac{c}{4\pi}\frac{1}{r_e}H \end{aligned}, \qquad (4.14)$$

are the “imaginary” currents, in which $\omega = \frac{2m_e c^2}{\hbar}$, and $r_e = \frac{\hbar}{2m_e c}$ is the radius of twirling of EM wave (and it is also the half of Compton wavelength of the electron). As we see the equations (4.13’) and (4.13’’) are Maxwell equations with imaginary electric and magnetic currents. As it is known the existence of the magnetic current $\vec{j}^m$ doesn't contradict to the quantum theory (see the Dirac theory of the magnetic monopole (Dirac, 1931)). In our case of the plane polarized wave (see figs. 2 and 3) the magnetic currents are equal to zero.

Thus, the equations (4.11) are Maxwell equations with imaginary tangential currents and simultaneously they are the Dirac equation of electron.

5.0. Analysis of the free electron equation solution from EM point of view

In accordance with the above results the electromagnetic form of the solution of the Dirac free electron equation must be a twirled electromagnetic wave.

If this supposition is actually correct, for the y-direction photon two solutions must exist:

1) for the wave, twirled around the OZ-axis

$$^{oz}\psi = \begin{pmatrix} E_x \\ 0 \\ 0 \\ iH_z \end{pmatrix} = \begin{pmatrix} \psi_1 \\ 0 \\ 0 \\ \psi_4 \end{pmatrix}, \tag{5.1}$$

and 2) for the wave, twirled around the OX-axis

$$^{ox}\psi = \begin{pmatrix} 0 \\ E_z \\ iH_x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \psi_2 \\ \psi_3 \\ 0 \end{pmatrix}, \tag{5.2}$$

The ψ- functions (5.1) and (5.2) as the solutions of the equations (4.11) must have the same expressions as the Dirac electron theory solutions (Schiff, 1955). Let us analyze the Dirac electron theory solutions from CWED point of view.

It is known (Schiff, 1955) that the solution of the Dirac free electron equation (2.1) has the form of the plane wave:

$$\psi_j = B_j \exp\left(-\frac{i}{\hbar}(\varepsilon\, t - \vec{p}\vec{r}) \right), \tag{5.3}$$

where $j = 1,\ 2,\ 3,\ 4$; $B_j = b_j e^{i\phi}$; the amplitudes b_j are the numbers and ϕ is the initial wave phase. The functions (5.3) are the eigenfunctions of the energy-momentum operators, where ε and $\vec{p}$ are the energy-momentum eigenvalues. Here for each $\vec{p}$, the energy ε has either positive or negative values according to the energy-momentum conservation law equation $\varepsilon_{\pm} = \pm\sqrt{c^2\vec{p}^2 + m_e^2 c^4}$.

For ε_+ we have two linear-independent set of four orthogonal normalizing amplitudes:

$$1)\ B_1 = -\frac{cp_z}{\varepsilon_+ + m_e c^2},\quad B_2 = -\frac{c(p_x + ip_y)}{\varepsilon_+ + m_e c^2},\quad B_3 = 1,\quad B_4 = 0, \tag{5.4}$$

$$2)\ B_1 = -\frac{c\left(p_x - ip_y\right)}{\varepsilon_+ + m_e c^2},\ \ B_2 = \frac{cp_z}{\varepsilon_+ + m_e c^2},\ \ B_3 = 0,\ \ B_4 = 1, \qquad (5.5)$$

and accordingly for ε_- :

$$3)\ B_1 = 1,\ \ B_2 = 0,\ \ B_3 = \frac{cp_z}{-\varepsilon_- + m_{ee} c^2},\ \ B_4 = \frac{c\left(p_x + ip_y\right)}{-\varepsilon_- + m_e c^2}, \qquad (5.6)$$

$$4)\ B_1 = 0,\ \ B_2 = 1,\ \ B_3 = \frac{c\left(p_x - ip_y\right)}{-\varepsilon_- + m_e c^2},\ \ B_4 = -\frac{cp_z}{-\varepsilon_- + m_e c^2}, \qquad (5.7)$$

Each of these four solutions (Schiff, 1955) can be normalized by its multiplication by normalization factor:

$$\kappa = \left[1 + \frac{c^2 \vec{p}^{\,2}}{\left(\varepsilon_+ + m_e c^2\right)^2}\right]^{-\frac{1}{2}},$$

which gives $\psi^+ \psi \ = 1$

Let's discuss these results.

1) The existing of two linear independent solutions corresponds with two independent orientations of the electromagnetic wave vectors and gives the unique logic explanation for this fact.

2) Since $\psi = \psi(y)$, we have $p_x = p_z = 0, \quad p_y = m_e c$ and for the field vectors we obtain: from (4.4) and (4.5) for "positive" energy

$$B_+^{(1)} = \begin{pmatrix} 0 \\ b_2 \\ b_3 \\ 0 \end{pmatrix} \cdot e^{i\phi},\ \ B_+^{(2)} = \begin{pmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{pmatrix} \cdot e^{i\phi}, \qquad (5.8)$$

and from (4.6) and (4.7) for "negative" energy:

$$B_-^{(1)} = \begin{pmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{pmatrix} \cdot e^{i\phi},\ \ B_-^{(2)} = \begin{pmatrix} 0 \\ b_2 \\ b_3 \\ 0 \end{pmatrix} \cdot e^{i\phi}, \qquad (5.9)$$

which exactly correspond to (5.1) and (5.2).

3) Calculate the correlations between the components of the field vectors. Putting $\phi = \frac{\pi}{2}$ for $\varepsilon_+ = m_e c^2$ and $\varepsilon_- = -m_e c^2$ we obtain correspondingly:

$$B_+^{(1)} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ i \cdot 1 \\ 0 \end{pmatrix}, \quad B_+^{(2)} = \begin{pmatrix} -\frac{1}{2} \\ 0 \\ 0 \\ i \cdot 1 \end{pmatrix}, \tag{5.10}$$

$$B_-^{(1)} = \begin{pmatrix} i \cdot 1 \\ 0 \\ 0 \\ -\frac{1}{2} \end{pmatrix}, \quad B_-^{(2)} = \begin{pmatrix} 0 \\ i \cdot 1 \\ \frac{1}{2} \\ 0 \end{pmatrix}, \tag{5.11}$$

Obviously the imaginary unit in these solutions indicates that the field vectors $\vec{E}$ and $\vec{H}$ are mutually orthogonal.

Also we see that the electric field amplitude is two times less, than the magnetic field amplitude. This fact shows that the electromagnetic field's values, which correspond to the Dirac equation solution, are different contrary to fields of the linear wave of the Maxwell theory, where $\vec{E} = \vec{H}$. (It can be shown that this result provides the electron stability).

4) It is easy to show that the electromagnetic form of the solution of the Dirac equation is the standing wave. Really in case of the circle-twirled wave we have $\vec{p} \perp \vec{r}$ and therefore $\vec{p} \cdot \vec{r} = 0$; then instead (4.3) we obtain:

$$\psi_j = b_j \exp\left(-\frac{i}{\hbar}\varepsilon\, t\right), \tag{5.12}$$

5) According with the Euler formula $e^{i\varphi} = \cos\varphi + i\sin\varphi$ the solution of the Dirac equation (5.12) describes a circle, as it corresponds to our theory.

6) Let's calculate the normalization factor, substituting: $p = m_e c, \varepsilon = m_e c^2$:

$$\kappa = \left(\frac{5}{4}\right)^{-\frac{1}{2}}, \tag{5.13}$$

and compare it with normalization factor, which is received from the electromagnetic representation of the theory. In view of that the electric field is twice less of magnetic field, the energy density of twirled semi-photon will be equal:

$$W_{s-ph} = \frac{1}{8\pi}\left(E_{s-ph}^2 + H_{s-ph}^2\right) = \frac{1}{8\pi}\left[\left(\frac{1}{2}H_{s-ph}{}^2\right) + H_{s-ph}^2\right] = \frac{1}{8\pi}\frac{5}{4}H_{s-ph}^2, \tag{5.14}$$

Using non-normalized expression for the wave function:

$$\psi_j = B_0 B_j e^{i\left(\vec{k}\vec{r}-\omega t\right)} = B_0 \begin{pmatrix} 0 \\ i\dfrac{1}{2} \\ 1 \\ 0 \end{pmatrix} e^{i\left(\vec{k}\vec{r}-\omega t\right)}, \tag{5.15}$$

(where B_0 is some constant, generally dimensional), and using also the Hermitian-conjugate function:

$$\psi_j^+ = B_0 B_j^+ e^{-i\left(\vec{k}\vec{r}-\omega t\right)} = B_0 \begin{pmatrix} 0 & -i\dfrac{1}{2} & 1 & 0 \end{pmatrix} e^{-i\left(\vec{k}\vec{r}-\omega t\right)}, \tag{5.16}$$

for field energy we will receive the following expression:

$$W = \frac{1}{8\pi}\psi_j^+\psi_j = \frac{1}{8\pi}\cdot\frac{5}{4}\cdot B_0^2\,, \tag{5.17}$$

which precisely corresponds to the quantum theory result.

6.0. Particularities of wave function of electron equation

As is known the fields of a photon are vectors, transforming as elements of group (O3). The spinor fields of the Dirac equation are transformed as elements of group (SU2). As it is shown by L.H. Ryder (Ryder, 1987) and others, two spinor transformations correspond to one transformation of a vector. For this reason the spinors are also named "semi-vectors" or "tensors of half rank" (Goenner, 2004; Sokolov & Ivanenko, 1952).

From above following that the twirling and breaking of the twirled photon waves corresponds to transition from usual linear Maxwell equation to the EM curvilinear wave equation with an imaginary tangential currents (i.e. to the EM Dirac equation). Obviously, the transformation properties of electromagnetic fields at this transition change. Just as the wave functions of the Dirac equation (i.e. spinors) submit to transformations of group (SU2), the semi-photon fields must submit to the same transformations.

Let us try now to specify the differences between electromagnetic fields $\{\mathrm{E}'_x, \mathrm{E}'_z, \mathrm{H}'_x, \mathrm{H}'_z\}$ of the Ψ-function of twirled photon and electromagnetic fields $\{E_x, E_z, H_x, H_z\}$ of ψ-function of twirled semi-photon. Taking into account that we have the same mathematical equations both for the CWED Dirac equation and the Dirac electron equation, we can affirm that these transformation features coincide with the same features of the spinor (Ryder, 1987; Gottfried & Weisskopf, 1984).

The spinor invariant transformation has the form:

$$\psi' = U\psi\,, \tag{6.1}$$

where the operator of transformation is entered as follows:

$$U\left(\vec{n}\theta\right)=\cos\frac{1}{2}\theta-i\vec{n}\cdot\vec{\sigma}'\sin\frac{1}{2}\theta\,, \tag{6.2}$$

where $\vec{n}$ is the unit vector of an axis, θ is a rotation angle around this axis and $\vec{\sigma}'=\left(\sigma_x',\sigma_y',\sigma_z'\right)$ is the spin vector.

The rotation matrix (6.2) possesses a remarkable property. If the rotation occurs on the angle $\theta=2\pi$ around any axis (therefore occurs the returning to the initial system of reference) we find, that $U=-1$, instead of $U=1$ as it was possible to expect. In other words, the state vector of system with spin half in usual three-dimensional space has two-valuedness and passes to itself only after turn to the angle 4π.

This result can be explained only if we suppose that the *EM electron is the twirled half-period of a twirled photon particle, and therefore needs to be rotated twice to return to the initial state.* In other words, the twirled semi-photon is the twirled half-period of the photon.

Taking into account the above results the solution of the EM electron equation (i.e. Dirac equation in the EM form) we can name "electromagnetic spinor". In other words the electromagnetic spinor is the semi-period of twirling EM wave. Thus, the transformation of the “linear” electromagnetic wave into curvilinear wave and its symmetry breaking produces the electromagnetic spinors.

7.0. Electromagnetic Non-linear Electron Equation and its Lagrangian

7.1. The EM nonlinear electron equation

Obviously the curvylinearity of photon or semi-photon motion must be described by non-linear equation. From this it follows that the CWED equation of EM electron must be the non-linear field equation. Let us find it electromagnetic and quantum forms.

The stability of twirled semi-photon is possible only by the semi-photon part’s self-action. Using (4.12) from (4.11) we will obtain the following non-linear equation:

$$\left[\hat{\alpha}_0\left(\hat{\varepsilon}-\varepsilon_{in}\right)+c\hat{\vec{\alpha}}\cdot\left(\hat{\vec{p}}-\vec{p}_{in}\right)\right]\psi=0\,, \tag{7.1}$$

where the inner energy and momentum can be expressed, using the inner energy density U and momentum density $\vec{g}$ (or Poynting vector $\vec{S}$) of EM wave:

$$\varepsilon_{in}=\int_0^\tau U\;d\tau=\frac{1}{8\pi}\int_0^\tau\left(\vec{E}^2+\vec{H}^2\right)\,d\tau\,, \tag{7.2}$$

$$\vec{p}_{in} = \int_0^\tau \vec{g}\ d\tau = \frac{1}{c^2}\int_0^\tau \vec{S}\ d\tau \ = \frac{1}{4\pi}\int_0^\tau \left[\vec{E}\times\vec{H}\right]\ d\tau\,, \tag{7.3}$$

putting the upper limit τ to be variable.

Substituting of the expression (7.2) and (7.3) to the EM electron equation, we obtain *the non-linear integral-differential equation, which is, as we suppose, the searched form of the non-linear equation*, which describes the EM-electron in both electromagnetic and concurrent quantum forms.

To show, that the equation (7.1) can actually pretends to the role of the equation of non-linear electrodynamics of the electron EM particle, we find its approximate quantum form.

Using EM form of ψ - function, it is easy to prove that the quantum forms of U and $\vec{S}$ are:

$$U \ = \frac{1}{8\pi}\ \psi^{+}\hat{\alpha}_0\psi\,, \tag{7.4}$$

$$\vec{S} \ = -\frac{c}{8\pi}\ \psi^{+}\hat{\vec{\alpha}}\ \psi = c^2\vec{g}\,, \tag{7.5}$$

Taking into account that the free electron Dirac equation solution is the plane wave:

$$\psi = \psi_0 \exp\left[i\left(\omega\ t - ky\right)\right], \tag{7.6}$$

we can write (7.2) and (7.3) in the next approximate form:

$$\varepsilon_p = U\ \Delta\tau \ = \frac{\Delta\tau}{8\pi}\psi^{+}\hat{\alpha}_0\psi\,, \tag{7.7}$$

$$\vec{p}_p = \vec{g}\ \Delta\tau = \frac{1}{c^2}\vec{S}\ \Delta\tau = -\frac{\Delta\tau}{8\pi\ c}\ \psi^{+}\hat{\vec{\alpha}}\ \psi\,, \tag{7.8}$$

where $\Delta\tau$ is the volume, which contain the main part of the twirled semi-photon energy. Then the approximate form of the equation (7.3) will be following:

$$\frac{\partial\ \psi}{\partial\ t} - c\hat{\vec{\alpha}}\vec{\nabla}\,\psi + i\,\frac{\Delta\tau}{8\pi\ c}\left(\psi^{+}\hat{\alpha}_0\psi - \hat{\vec{\alpha}}\psi^{+}\hat{\vec{\alpha}}\psi\right)\ \psi = 0\,, \tag{7.9}$$

It is not difficult to see that the equation (7.9) is the non-linear equation of the same type as non-linear Heisenberg equation(Heisenberg, 1966; Paper translation collection, 1959):

$$\gamma_\mu \frac{\partial \psi}{\partial x_\mu} + \frac{1}{2} l \left[\gamma_\mu \psi \left(\overline{\psi} \gamma_\mu \psi \right) + \gamma_\mu \gamma_5 \psi \left(\overline{\psi} \gamma_\mu \gamma_5 \psi \right) \right] = 0 \,, \tag{7.10}$$

if instead of α-set Dirac matrices we will use γ-set matrices (here l is some positive constant). The non-linear equation (7.10) was postulated and investigated by Heisenberg et. al. as the unitary quantum field theory equation. Contrary to the last one, the equation (7.9) is obtained by logical and correct way and the self-action constant l appeared in (7.9) automatically. As it is known in the framework of this non-linear unitary field theory some substantial achievements were made.

7.2. The Lagrangian of the nonlinear electron theory

The Lagrangian of the Dirac electron theory of linear type in quantum form is (Schiff, 1955):

$$L_D = \psi^+ \left(\hat{\varepsilon} + c \hat{\vec{\alpha}}\ \hat{\vec{p}} + \hat{\beta} m_e c^2 \right) \psi, \tag{7.11}$$

It is not difficult to find its electromagnetic form:

$$L_D = \frac{\partial\, U}{\partial\, t} + div\ \vec{S} - i \frac{\omega}{8\pi} \left(\vec{E}^2 - \vec{H}^2 \right), \tag{7.12}$$

(Note that in the case of the variation procedure we must distinguish the complex conjugate field vectors $\vec{E}^*$, $\vec{H}^*$ and $\vec{E}$, $\vec{H}$).

The Lagrangian of non-linear theory is not difficult to obtain from the Lagrangian (7.11) using the method by which we found the nonlinear equation. By substituting (5.1) we obtain:

$$L_N = \psi^+ \left(\hat{\varepsilon} \ - c \hat{\vec{\alpha}} \cdot \hat{\vec{p}} \right) \psi + \psi^+ \left(\varepsilon_{in} - c \hat{\vec{\alpha}} \cdot \vec{p}_{in} \right) \psi \,, \tag{7.13}$$

We suppose that the expression (7.13) represents the common form of the Lagrangian of the non-linear electron theory. In order to compare (7.13) with the known results of classical and quantum physics let us find the approximate electromagnetic and quantum forms of this Lagrangian.

Using (7.7) and (7.8) we can represent (7.11) in the following quantum form:

$$L_N = i\hbar \left[\frac{\partial}{\partial\, t} \left[\frac{1}{2} \left(\psi^+ \psi \right) \right] - c\, div \left(\psi^+ \hat{\vec{\alpha}} \psi \right) \right] + \frac{\Delta \tau}{8\pi} \left[\left(\psi^+ \psi \right)^2 - \left(\psi^+ \hat{\vec{\alpha}} \psi \right)^2 \right], \tag{7.14}$$

To obtain the EM form of (7.14) we initially pass on to normalized ψ-function, using the expression $L'_N = \dfrac{1}{8\pi\ mc^2} L_N$. Then we transform (7.13), using equations (7.4) and (7.5), and obtain from (7.14) the following approximate electromagnetic form:

$$L'_N = i\frac{\hbar}{2m_e}\left(\frac{1}{c^2}\frac{\partial U}{\partial t} + div\ \vec{g}\right) + \frac{\Delta\tau}{m_e c^2}\left(U^2 - c^2\vec{g}^2\right), \qquad (7.15)$$

It is not difficult to transform the second term, using the known identity of electrodynamics:

$$(8\pi)^2\left(U^2 - c^2\vec{g}^2\right) = \left(\vec{E}^2 + \vec{H}^2\right)^2 - 4\left(\vec{E}\times\vec{H}\right)^2 = \left(\vec{E}^2 - \vec{H}^2\right)^2 + 4\left(\vec{E}\cdot\vec{H}\right)^2, \qquad (7.16)$$

Thus, taking into account that $L_D = 0$ and using (7.12) and (7.16), we obtain from (7.15) the following expression:

$$L'_N = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{H}^2\right) + \frac{\Delta\tau}{(8\pi)^2 m_e c^2}\left[\left(\vec{E}^2 - \vec{H}^2\right)^2 + 4\ \left(\vec{E}\cdot\vec{H}\right)^2\right], \qquad (7.17)$$

As we see, the approximate form of the Lagrangian of the nonlinear equation of the twirled electromagnetic wave contains only the invariants of the Maxwell theory and is similar to the known Lagrangian of the photon-photon interaction (Akhiezer and Berestetskii, 1965).

Let us now analyze the quantum form of the Lagrangian density (7.17). The equation (7.12) can be written in the form:

$$L_Q = \psi^+\hat{\alpha}_\mu\partial_\mu\psi\ + \frac{\Delta\tau}{8\pi}\left[\left(\psi^+\hat{\alpha}_0\psi\right)^2 - \left(\psi^+\hat{\vec{\alpha}}\ \psi\right)^2\right], \qquad (7.18)$$

It is not difficult to see that the electrodynamics correlation (7.16) in quantum form has the form of the known Fierz identity (Cheng and Li, 1984; 2000):

$$\left(\psi^+\hat{\alpha}_0\psi\right)^2 - \left(\psi^+\hat{\vec{\alpha}}\ \psi\right)^2 = \left(\psi^+\hat{\alpha}_4\psi\right)^2 + \left(\psi^+\hat{\alpha}_5\psi\right)^2, \qquad (7.19)$$

Using (7.19) from (7.18) we obtain:

$$L_Q = \psi^+\hat{\alpha}_\mu\partial_\mu\psi\ + \frac{\Delta\tau}{8\pi}\left[\left(\psi^+\hat{\alpha}_4\psi\right)^2 - \left(\psi^+\hat{\alpha}_5\psi\right)^2\right], \qquad (7.20)$$

The Lagrangian (7.20) coincides with the Nambu and Jona-Lasinio Lagrangian (Nambu and Jona-Lasinio, 1961; 1961a), which is the Lagrangian density of the relativistic superconductivity theory. As it is known this Lagrangian density is used for the solution of the problem of the elementary particles mass appearance by the mechanism of the vacuum symmetry spontaneous breakdown (it corresponds also to the Cooper's pair production process in the superconductivity theory).

Note again that in our theory, the breakdown of symmetry also takes place when a mass of particles appears within twirling and breaking of photon.

8.0. About peculiarities of CWED as the non-linear theory

It is not difficult to see that CWED disclose two types of non-linearity. The first type is connected with postulate 6 of CWED about the motion of EM wave along curvilinear closed trajectories. The curvilinearity, as a deviation from linearity, is possible to consider as one of kinds of non-linearity. But in our case these non-linear trajectories concern to concrete kind: they are created and described by harmonic functions and by their superpositions. It allows to describe this non-linearity by the linear equations.

Really, the motion along a circle can be presented as the sum of two linear harmonic oscillations. The sum of greater number of oscillations leads to the multiform (including, spatial) curvilinear trajectories, known as Lissajous figures. Apparently, in this connection all these non-linearities are conveniently and simply described by complex functions (more detail see chapter 8). It is possible to assume that the Fourier apparatus of the analysis and synthesis of functions reflects such opportunity of the linear description of curves, which can be described by the sum of the linear harmonic oscillations.

In this case it is possible in the existence of Fourier theory to see the reflection of the reality, described by CWED. Since the Fourier theory can be used only in the case of linear functions, obviously, this "harmonic curvilinearity" allows in these conditions to consider the CWED to be the linear theory, i.e. the theory, in which as well as in the quantum field theory, the principle of superposition is strictly carried out.

But, on the other hand, as we saw, the twirling of EM waves results also in other type of non-linearity. Really, we deal here not only with trajectories, but with the fields, which "are attached" to this trajectory by strictly defined manner. During formation of EM particles, i.e. as a result of bending of trajectory of an EM wave, inside of its volume the field configuration varies. This enters into the equations the non-linear terms, which are presented neither in classical electrodynamics, nor in the linear quantum field theory. The splitting up of the twirled photon into two twirled half-period even more complicates this picture. Thus, strictly speaking, inside of a particle operates the non-linear field theory and apparently the principle of superposition should here not have place.